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# Glafka-2004: Categorical Quantum Gravity[1]

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A synopsis-*cum*-update of work in the past half-decade or so on applying the algebraico-categorical concepts, technology and general philosophy of Abstract Differential Geometry (ADG) to various issues in current classical and quantum gravity research is presented. The exposition is mainly discursive, with conceptual, interpretational and philosophical matters emphasized throughout, while their formal technical-mathematical underpinnings have been left to the original papers. The general position is assumed that Quantum Gravity is in need of a new mathematical, novel physical concepts and principles introducing, framework in which old and current problems can be reformulated, readdressed and potentially retackled afresh. It is suggested that ADG can qualify as such a theoretical framework.

**KEY WORDS:** general relativity; quantum gravity; abstract differential geometry; sheaf theory; category theory; topos theory.

## 1. PROLOGUE: GENERAL MOTIVATIONAL REMARKS

Quantum Gravity (QG) has as many facets as there are approaches to it. There is no unanimous agreement on what QG 'really' is—what are its central questions, its main aims, its basic problems, or what ought to be ultimately resolved; hence the current 'zoo' of approaches to it. There certainly is overlap between the concepts, the mathematical techniques and the basic aims of the various approaches, but the very fact that there are so many different routes to such a supposedly fundamental quest betrays more our ignorance rather than our resourcefulness about what QG 'truly' stands for, or at least about how it should be 'properly' addressed and approached.

*Prima facie*, the danger that goes hand in hand with the said proliferation of approaches to QG observed lately is that the *aufbau* of such a theory may

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eventually degenerate into the erection of some kind of Babel Tower, where workers working on each individual approach, just by virtue of the big number of different, simultaneously developing, schemes (with the concomitant development of 'idiosyncratic' conceptual and technical jargon, as well as approach-specific mathematical techniques), may find it difficult to communicate with each other. As a result, like the mutually isolated seagull populations of the Galapagos islands that Charles Darwin came across, the various approaches may eventually cease to be able to cross-breed and the workers will become 'alienated' from each other—*i.e.*, they will not be able to communicate, let alone to fruitfully interact, check or cross-fertilize each other's ideas and results. Thus, the QG vision shall inevitably become disorientated and fragmented; and what's worse, perhaps irreversibly so. It will then be hard to believe that all these different workers and their ventures do indeed have a common goal (:QG), even if they nominally say so (*e.g.*, in conferences!).

Of course, there is that general feeling, ever since the inception and advent of General Relativity (GR) and subsequently of Quantum Mechanics (QM), that QG ought to be a coherent amalgamation of those two pillar theories of 20th century theoretical physics. Perhaps one of the two theories (or even both!) may have to undergo significant modifications in order for QG to emerge as a consistent 'unison-by-alteration' of the two. On the other hand, the gut feeling of many (if not of most) workers in the field is that, no matter how advanced and sophisticated our technical (:mathematical) machinery is, we lack the proper conceptual-physical questions that will open the Pandora's box of QG. It may well be that the fancy maths get in the way of the simple fundamental questions we need to come up with in order to crack the QG 'code'. We may be rushing, primarily dazed by past successes of our mathematical panoply, to give intricate and complex mathematical answers to simple, yet profound, physical questions that have not been well posed, or even asked(!), yet. Fittingly here, Woody Allen's

> "I have an answer, can somebody please tell me the question?"

springs to mind. Time and again the history of the development of theoretical physics has taught us that in the end, *Nature invariably outsmarts our maths* no matter how sophisticated and clever they may be, while our own knowledge is not only insignificant compared to Her wisdom, but also many times it sabotages the very path that we are trying to pave towards the fundamental physical questions. For, very often, (mathematical) knowledge inhibits (physical) intuition and imagination.

Or perhaps, in a promethean sense opposite to that above, it may be that

> *we are not adventurous and 'iconoclastic' enough in our theory-making enterprizes as well as in the mathematical means that we employ so as to take the 'necessary'*

> *risks to look at the QG problem afresh*[4]—*e.g.*, by creating new theoretical concepts, new mathematical tools and techniques, as well as a novel way of philosophizing about them.

In keeping with the 'zoological' metaphor above,

> so far the attempts to bring together GR and QM to a cogent (*i.e.*, a conceptually sound, mathematically consistent, as well as calculationally finite) QG, seem to this author to be like *trying to cross a parrot with a hyena: so that it (:QG) can tell us what it is laughing about*.

All in all, it may well be the case that the QG riddle has been with us for well over half a century now, stubbornly resisting (re)solution and embarrassingly eluding all our sophisticated mathematical means of description, because we insist on applying and trying to marry the 'old' physical concepts and maths—which, let it be appreciated here, have proven to be of great import in formulating separately the ever so successful and experimentally vindicated GR and QM—to the virtually unknown realm of QG.[5] The following 'words of caution' by Albert Einstein (1990) are very pertinent to this discussion:

> "... Concepts which have proven useful for ordering things easily assume so great an authority over us, that we forget their terrestrial origin and accept them as unalterable facts. They then become labelled as 'conceptual necessities', 'a priori situations', etc. The road of scientific progress is frequently blocked for long periods by such errors. It is therefore not just an idle game to exercise our ability to analyse familiar concepts, and to demonstrate the conditions on which their justification and usefulness depend, and the way in which these developed, little by little... " (1916).

In the present paper we take sides more with the second alternative above, namely, that a new theoretical/mathematical framework—one that comes equipped with new concepts and principles, and it is thus potentially able to cast new light on old ones, as well as to generate new physical questions—is needed to readdress, reformulate and possibly retackle *afresh* certain caustic, persistently problematic issues in current QG research. The framework we have in mind is Mallios' purely algebraico-categorical (:sheaf-theoretic) Abstract Differential Geometry (ADG) (Mallios, 1998a,b, 2005c), while the account that follows is a semantic, conceptual and philosophical distillation-*cum*-update of results (and their related aftermath) of a series of applications of ADG to gravity[6] in the past half-decade or so (Mallios, 1998b, 2001, 2002, 2003, 2005a,b; Mallios and Raptis, 2001, 2002, 2003, 2005;

[4] See this author's introduction to this volume.

[5] This 'palindromic' thesis between *too much* and *not enough* maths for QG, simply reflects the mean, neutral position of ignorance, ambivalence and uncertainty of this author about these matters. See concluding section.

[6] In the sequel, gravity (classical or quantum), formulated ADG-theoretically, will be coined '*ADG-gravity*' (Mallios and Raptis, 2005; Raptis, 2005a,b).

Mallios and Rosinger, 1999, 2001, 2002; Raptis, 2005a,b). Further details about formal-technical (:mathematical) terms and results are left to those original papers.

After this introduction, the paper unfolds in three sections, as follows: in the next section we give a brief *résumé* of the principal didactics, as well as the basic physical concepts, semantics and hermeneutics of ADG. The section that follows it addresses certain important current classical and quantum gravity issues under the prism of the background spacetime manifoldless ADG, and it ends with a brief discussion of current and near future developments of the theory along topos and more general category-theoretic lines. The paper closes by continuing the way it started; *i.e.*, by making general remarks on the significance and import of a new mathematical-theoretical framework (such as ADG) in current and future QG research.

## 2. THE BASIC TENETS AND DIDACTICS OF ADG

ADG, we have learned both from theory and from numerous applications, is a way of doing differential geometry *purely algebraically* (:sheaf-theoretically), without using any notion of smoothness in the usual sense of Classical Differential Geometry (CDG)[7]—*i.e.*, without employing a base geometrical differential manifold. *In summa*, ADG is a Calculus-free, entirely algebraic, background manifoldless theoretical framework of differential geometry (Mallios, 1998a,b, 2005c).

At the basis of ADG lies the notion of **K**-*algebraized space* ($\mathbf{K} = \mathbf{R}, \mathbf{C}$), by which one means an in principle arbitrary base topological space $X$, carrying a sheaf $\mathbf{A}$ of (commutative) $\mathbb{K}$-algebras ($\mathbb{K} = \mathbb{R}, \mathbb{C}$) called the *structure sheaf of generalized arithmetics or coordinates*. A family $\mathcal{U}$ of open subsets $U$ of $X$ covering it is called a *system of local open gauges*, while our generalized local measurements (of coordinates) relative to $\mathcal{U}$ are modelled after the local sections of $\mathbf{A}$, $\mathbf{A}(U) \equiv \Gamma(\mathcal{U} \ni U, \mathbf{A})$. With $\mathbf{A}$ in hand, a *vector sheaf* $\boldsymbol{\mathcal{E}}$ of rank $n$ is a sheaf of vector spaces of dimensionality $n$ that is locally expressible as a finite power (:Whitney sum) of $\mathbf{A}$: $\boldsymbol{\mathcal{E}}(U) \simeq \mathbf{A}^n(U)$. By a *local gauge frame* $e^U$ ($\mathcal{U} \ni U \subset X$), one means an $n$-tuple $(e_1, e_2, \ldots, e_n)$ of local sections of $\boldsymbol{\mathcal{E}}$ providing a basis for the vector spaces inhabiting its stalks. Let it be stressed here that the role of $X$ is just as a 'surrogate scaffolding', which serves as a substrate for the sheaf-theoretic localization of the objects living in the stalks of the vector and algebra sheaves involved. $X$ has no physical significance, as we shall argue below.

One realizes from the beginning how important $\mathbf{A}$ is in the theory. We take it almost axiomatically that

> *there is no 'geometry' without measurement, and no measurement without a difference— i.e.*, what we measure is always differences or changes in some 'measurable' quantities

[7] In the sequel, the names Differential Calculus (or simply Calculus) and Analysis shall be regarded as synonyms to the CDG of smooth manifolds.

> (*e.g.*, coordinates),[8] the variability of which is being secured in our scheme by the fact that, in the case of coordinates, **A** is a *sheaf*.

Indeed, the notion of sheaf is intimately entwined with that of localization, which physically may be thought of as the *act of gauging physical quantities*, which in turn essentially promotes them to (dynamically) variable entities. The bottom-line of all this is that

> the algebras in **A** are *differential* algebras—*i.e.*, they are able to provide us with some kind of differential operator, via which then we represent the said (dynamical) changes (:differences).

In turn, we assume that all the 'observables' (:measurable dynamically variable physical quantities) in our theory can always be expressed in terms of **A** (*e.g.*, as $\otimes_{\mathbf{A}}$-tensors).[9] In a subtle sense,

> from the ADG-theoretic perspective *all differential geometry boils down to the* **A** *that we choose to use up-front in the theory's aufbau.*

Parenthetically, but in the same line of thought, we would like to answer briefly to Shing-Shen Chern's philosophical pondering in Chern (1990):

> ". . . A mystery is the role of differentiation. The analytic method is most effective when the functions involved are smooth. Hence I wish to quote a philosophical question posed by Clifford Taubes:[10] Do humans really take derivatives? Can they tell the difference?. . . "

by holding that *humans do indeed differentiate* (and they can 'really' tell the difference!) *insofar as they can measure*.[11] From the ADG-theoretic vantage, they can indeed assume different **A**s, provided of course these structure sheaves of generalized arithmetics (:coordinates or measurements) furnish them with a

[8] *En passant*, let it be stressed here that it is *we* the theorists that declare and determine up-front what is measurable when we build up our theories. In this sense, theory and observation are closely tied to each other (in Greek, '*theory*', *viz.*, '$\theta\epsilon\omega\rho\acute{\iota}\alpha$', means 'a way of *looking* at things'). In a deep sense, we see what *we* want to look at (even in the mind's eye). This also recalls Einstein's advice to Heisenberg that, apart from the fact that a theory cannot be built solely on observable quantities, "*it is the theory that determines what can be observed, not the other way round*" (Heisenberg, 1989). *In toto*, 'geometry' is a creature of the theorist, since it is effectively a mathematical encodement of and sums up all her observations (:'measurements'). However, as Einstein advised above, in a physical theory not all entities are 'geometrical' (:'observable' or 'measurable'). (See remarks in the sequel about the principal notion of connection $\mathcal{D}$ in ADG and ADG-gravity).

[9] $\otimes_{\mathbf{A}}$ is the homological tensor product functor.

[10] The reference given here is Taubes (1984).

[11] To be precise, in Taubes (1984) Taubes was talking about so-called *inequivalent differential structures* that a manifold can admit (*e.g.*, *à la* John Milnor). In anticipation of the basic ADG-didactics that follow below, our reply here has a slightly different sense, pertaining to Chern's mentioning that the most effective method (of differentiating) is that of Analysis, via smooth manifolds.

differential operator (*viz*. connection) $\partial$. This discussion brings us to the central notion of ADG.

The neuralgic concept of ADG, as befits any scheme that aspires to qualify as a theory of *differential* geometry proper, is that of *connection* $\mathcal{D}$ (*alias*, generalized differential $\partial$). $\partial$ (or $\mathcal{D}$) is categorically defined as a **K**-linear, Leibnizian *sheaf morphism* between **A** (or $\boldsymbol{\mathcal{E}}$), and a sheaf $\Omega$ of **A**-modules of differential form-like entities being the ADG-analogues of the smooth differential forms encountered in CDG. The connections in ADG are fittingly coined **A**-connections, since **A** is the 'source' of the differential operator $\partial$ (or equivalently, $\boldsymbol{\mathcal{E}} \simeq_{\text{loc}} \mathbf{A}^n$ is the 'domain' of $\mathcal{D}$). In turn, by a *field* in ADG, one refers to the pair $(\boldsymbol{\mathcal{E}}, \mathcal{D})$, where $\boldsymbol{\mathcal{E}}$ is the carrier space of the connection $\mathcal{D}$.[12] The ADG-conception of $\partial$ and $\mathcal{D}$ is a Leibnizian (*i.e.*, relational, algebraic), not a Newtonian, one. That is, in ADG we obtain the differential (structure) from the algebraic relations (:structure) of the objects living in the stalks of the vector and algebra sheaves involved, and not from a background geometrical 'space(time)' continuum (:manifold), which 'cartesianly' mediates in our Calculus (ultimately, in our differential geometric calculations) in the guise of (smooth) coordinates as in the usual CDG of manifolds.

With $\partial$ and $\mathcal{D}$ in hand, we can then define the important notion of *curvature* $R$ of a connection $\mathcal{D}$, an **A**-metric $\rho$, torsion, and all the standard concepts and constructions of the (pseudo-)Riemannian geometry of GR; albeit, to stress it again, entirely algebraico-categorically, without using any background geometrical locally Euclidean (:manifold) space(time). $R$, like $\mathcal{D}$, is a sheaf morphism, but unlike its underlying connection which is only a **K**-morphism, it is an **A**-morphism (or $\otimes_{\mathbf{A}}$-tensor). The dynamical relations (:physical laws) between the observable physical quantities noted above are then expressed differential geometrically as differential equations proper. In other words,

> in ADG the laws of physics are categorically expressed as *equations between sheaf morphisms*,[13] such as the curvature of the connection.

In ADG-gravity in particular, the vacuum Einstein equations are formulated in terms of the Ricci scalar curvature $\mathcal{R}$ of a gravitational connection $\mathcal{D}$:[14]

$$\mathcal{R}(\boldsymbol{\mathcal{E}}) = 0 \tag{1}$$

[12] This definition of a field may be thought of as an abstraction and generalization of Yuri Manin's definition of an electromagnetic (:Maxwell) field as a connection on a line bundle (although in ADG we do not work with fiber bundles, but with sheaves, which are more 'flexible' and versatile structures).

[13] From this it follows what we noted earlier, namely, that the base arbitrary topological space $X$ plays absolutely no role in the physical dynamics in our theory.

[14] This is the only displayed mathematical expression in the present paper!

Perhaps the deepest observation one can make about (1) above is that it is an '**A**-*functorial*' expression. This means that the Einstein equations are expressed via the curvature of the connection (and not directly in terms of the connection itself!), which as noted above is an **A**-morphism (:an $\otimes_{\mathbf{A}}$-tensor). The gravitational field, in the guise of $R(\mathcal{D})$, 'sees through' and it is unaffected (*i.e.*, it remains 'invariant') by our generalized measurements in **A**. This is a *categorical* description of the ADG-analogue of the Principle of General Covariance (PGC) of GR, which group-theoretically may be represented by $\mathcal{A}ut\mathcal{E}$ as we shall note in the next section. In connection with the discussion around footnote 5 above, it is interesting to note that the principal entity in ADG-gravity, the gravitational connection $\mathcal{D}$, strictly speaking is *not* itself an 'observable'—*i.e.*, a *measurable* dynamical entity in the theory—as it is *not* a 'geometrical object' (:an **A**-morphism or $\otimes_{\mathbf{A}}$-tensor). However, its curvature $R(\mathcal{D})$ is an observable, and the vacuum Einstein equations (1) are expressed via it.[15] The moral here *vis-à-vis* Einstein's advice to Heisenberg in footnote 5, is that the central notion in ADG-gravity (and in ADG in general)—that of connection $\mathcal{D}$—is an 'unobservable' entity, as it eludes our generalized coordinates (:measurements) in **A**.

In turn, on the last observation above rests our generalized Principle of Field Realism (PFR), which is closely related to our categorical version of the PGC of GR noted earlier (:**A**-functoriality), and roughly it maintains that

> The ADG-gravitational field $\mathcal{D}$, and the field law (1) that it defines differential geometrically (:as a differential equation proper), remains unaffected (and the corresponding law 'invariant') by our 'subjective', arbitrary choices of **A**.

Einstein's words below, taken from his '*Time, Space, and Gravitation*' article in Einstein (1950) where he gives an account of how he arrived at the PGC of GR as 'invariance of the law of gravity under arbitrary coordinate transformations', are very relevant here:

> "... Must the independence of physical laws with regard to a system of coordinates be limited to systems of coordinates in uniform movement of translation with regard to one another? *What has nature to do with the coordinate systems that we propose and with their motions?*[16] Although it may be necessary for our descriptions of nature to employ systems of coordinates that we have selected arbitrarily, the choice should not be limited in any way so far as their state of motion is concerned[17] ... "

the subtle but important generalization of the PGC of GR by ADG-gravity culminating in the PFR above is that

[15] On this remark hinges the observation that $\mathcal{D}$ *is not a geometrical entity*; rather, it is an *algebraic* (:analytic) one. (See also Anastasios Mallios' contribution to this volume (Mallios, 2005b).)

[16] Our emphasis.

[17] Or perhaps better expressed, (the said arbitrary choice of any particular system of) coordinates should not affect in any way the dynamical equations (laws) of motion of the fields in focus.

> the field law of gravity remains unaffected (:'invariant') not only by arbitrary (:general) *smooth* coordinate transformations (*i.e.*, by general transformations of coordinates within the structure sheaf $\mathbf{A} \equiv \mathcal{C}^{\infty}_{M}$ chosen by the theorist/'observer'), but also by arbitrary changes of $\mathbf{A}$ itself.

In our work this last remark has been promoted to a principle, coined the Principle of Algebraic Relativity of Differentiability (PARD), and it maintains that

> no matter what $\mathbf{A}$ is chosen to furnish us with, and thus to geometrically represent (in $\mathcal{E}$), the gravitational field $\mathcal{D}$, the field law of gravity that the latter defines remains unaffected by it.

Thus, as a pun to Taubes' question that Chern was quoted as asking in the previous section, we can now retort: *the ADG-gravitational connection field is indifferent to different choices of differential algebras of generalized coordinates* $\mathbf{A}$ *that we employ to represent it (on* $\mathcal{E}$*).* For, to emulate Einstein's words above: *what has nature (here, the gravitational field law) to do with the* $\mathbf{A}$*s that we choose to geometrically represent (via* $\mathcal{E}$*) the (inherently algebraic) gravitational field* $\mathcal{D}$*?*

In closing this section, it must be stressed in view of the last remarks and footnote above that the generalized coordinates in $\mathbf{A}$, once they supply us with the differential geometric mechanism—*i.e.*, with the differential $\partial$ or the connection $\mathcal{D}$—they are effectively (*i.e.*, as far as the expression of the field law of gravity is concerned) 'discarded' as they have absolutely no physical significance, since the gravitational field dynamics (1) 'sees through' them (:it is $\mathbf{A}$-covariant, or $\mathbf{A}$-functorial). It took Einstein more than 7 years to appreciate the metric and hence the dynamical[18] insignificance of coordinates; albeit, the smooth base spacetime manifold (:$\mathbf{A}_X \equiv \mathcal{C}^{\infty}_{M}$) is invaluable in standard GR, if anything, in order to formulate the theory differential geometrically (*i.e.*, to model the dynamics after differential equations proper) (Kriele, 1999).

*In toto*, in GR too, the Einstein equations are generally covariant since they are formulated as differential equations between smooth, $\otimes_{\mathcal{C}^{\infty}_{M}}$-tensors. The subtle point here is that in the manifold and CDG-based GR, whenever a concrete calculation is made, the smooth coordinates are invoked and the background spacetime continuum provides us with a geometro-physical interpretation of the theory. That is, in GR, spacetime events and smooth spacetime intervals between them have a direct experimental meaning, as they are 'quantities' to be measured (:recall that $g_{\mu\nu}$ represents both the gravitational field and the spacetime chronogeometry). By contrast, in the purely algebraic ADG-gravity, there is *a priori* no need for a geometrical (smooth) spacetime interpretation of the theory.[19] Here is a challenging question for future physical applications of ADG:

[18] Since in GR *à la* Einstein, the metric $g_{\mu\nu}$ is the sole dynamical variable.

[19] This doing away with the smooth background geometrical spacetime manifold of ADG-gravity proves to be very important in both classical and quantum gravity current research as we shall argue in the next section.

*Can we relate the theory (:ADG-gravity) to experience directly from its purely algebraic underpinnings, without recourse to a background geometrical manifold representation and its associated spacetime interpretation?*[20]

## 3. IMPLICATIONS OF BACKGROUND SPACETIME MANIFOLDLESSNESS

In this section we outline the main 'aftermaths'—*i.e.*, the results following the application of the ADG-maths (pun intended)—of numerous applications of the base spacetime manifoldless ADG to gravity. To prevent the reader's distraction from repeated referencing within the text, the citations where all the results that follow can be found are Mallios (1998b, 2001, 2002, 2003, 2005a,b), Mallios and Raptis (2001, 2002, 2003, 2005), Mallios and Rosinger (1999, 2001, 2002), Raptis (2005a,b).

*ADG-gravity as pure gauge theory of the 3rd kind.*. ADG-gravity has been called '*pure gauge theory of the third kind*' due to the following three characteristic features:

- First, the sole dynamical variable in ADG-gravity is the **A**-connection $\mathcal{D}$. This is in contradistinction to the original second-order formalism of GR due to Einstein in which the sole dynamical variable is the spacetime metric $g_{\mu\nu}$ whose ten components represent the gravitational potentials, or even to the recent first-order Palatini-type of formalism due to Ashtekar in which two gravitational variables are involved—the tetrad field $e_\mu$ and the spin-Lorentzian connection $\mathcal{A}$.[21] Fittingly, the ADG-formulation of gravity has been called '*half-order formalism*', since only half the variables (namely, only the connection) of the first-order formalism are involved.
- Second, due to the manifest absence of a background geometrical smooth spacetime manifold $M$, there is no distinction between external (:spacetime) and internal (:gauge) symmetries. In ADG-gravity, the $\mathrm{Diff}(M)$ of external smooth spacetime symmetries, traditionally implementing the PGC in the manifold and, *in extenso*, the CDG-based GR, is replaced by $\mathcal{A}ut\boldsymbol{\mathcal{E}}$—the principal group sheaf of automorphisms of the ADG-gravitational field $(\boldsymbol{\mathcal{E}}, \mathcal{D})$. Of course, by virtue of the local isomorphism $\boldsymbol{\mathcal{E}}|_U \simeq \mathbf{A}^n$, $\mathcal{A}ut\boldsymbol{\mathcal{E}}$ assumes locally the more familiar form: $\mathcal{A}ut\boldsymbol{\mathcal{E}}|_U = \mathcal{GL}(n, \mathbf{A}(U))$—the group sheaf of general (generalized) coordinates' transformations. This is a Kleinian perspective on field geometry: the geometry of the field (:and

[20] This author is indebted to the referee of Raptis (2005a) for bringing him to ask this question with his acute remarks on the connection between ADG-gravity's doing away with coordinates and experiment.

[21] Let it be noted here that the smooth metric of the original 2nd-order formalism is still present 'in disguise' in Ashtekar's (1986) scheme, as $g_{\mu\nu}$ is effectively encoded in the *vierbein* $e_\mu$s.

concomitantly, of the law that it defines) is its automorphism group (:and concomitantly, the symmetries of the law that it defines).

- And third, from the above it follows that ADG-gravity is neither a gauge theory of the 1st kind (:global gauge symmetries, global gauge frames), nor one of the 2nd kind (:spacetime localized gauge symmetries, local gauge frames). There is no external, to the ADG-gravitational field ($\mathcal{E}$, $\mathcal{D}$), spacetime. The field is a dynamically autonomous entity, whose 'auto-symmetries' (:'self-invariances' of the law (1) that it defines) are encoded in $\mathcal{A}ut\mathcal{E}$. This makes the ADG-gravitational field an autonomous, '*external spacetime unconstrained gauge system*'. As a result, in ADG-gravity there is no distinction between external (:'spacetime') and internal (:'gauge') symmetries: all symmetries are 'esoteric' to the field, pure gauge ones.

In view of the above, the '*background smooth spacetime manifoldless half-order formalism*' of ADG-gravity may shed light on the outstanding problem of treating gravity as a gauge theory proper (Ivanenko and Sardanashvily, 1983)—a problem which is largely due to our persistently fallacious viewing of Diff($M$) as a gauge group proper (Weinstein, 1998).

In the absence of an external (:background) geometrical spacetime manifold $M$ and the autonomous conception of the gravitational field in ADG-gravity, we encounter no problems originating from $M$ and its Diff($M$) 'structure group'. On the other hand, the classical theory (GR), as well as various attempts to quantize it by retaining the base $M$ and hence the entire CDG-technology, do encounter such problems—one of them being the problem of regarding gravity as a gauge theory proper mentioned above. Let us discuss some more of them.

*The role of singularities in ADG-gravity.* The role of singularities in GR was well known and appreciated since the times of Einstein and Schwarzschild, but it got worked out and further clarified in the celebrated works of Hawking and Penrose in the late 60s/early 70s. Briefly, singularities are thought of as *loci* in the spacetime continuum where some physically important quantity grows without bound and, ultimately, the Einstein gravitational equations seem to break down. Given some generic conditions, the Einstein equations appear to 'predict' singularities—sites of their own destruction. This is pretty much the general aftermath of the manifold based Analysis of spacetime singularities (Clarke, 1993). In this Analysis (and this is the general consensus in gravitational physics), although singularities are pushed to the boundary of an otherwise regular spacetime manifold, they are regarded as being physically significant, in spite of Einstein's position to the contrary till the end of his life (Einstein, 1956):

> "... A field theory is not yet completely determined by the system of field equations. Should one admit the appearance of singularities?... *It is my opinion that singularities*

> *must be excluded. It does not seem reasonable to me to introduce into a continuum theory points (or lines etc.) for which the field equations do not hold*[22] ..."

In this line of thought however, few would doubt that the main culprit for the singularities of GR is the smooth base spacetime manifold which is *a priori* assumed in the theory, in the sense that every singularity is a pathology of a smooth function in $\mathcal{C}^{\infty}_{M}$—the sheaf of germs of smooth functions on $M$.[23] Moreover, the very PGC of GR, which is mathematically implemented via Diff$(M)$ as noted before, appears to come in conflict with the existence of gravitational singularities, which makes a precise definition of the latter perhaps the most problematic issue in GR (Clarke, 1993; Geroch, 1968).

By contrast, in the base spacetime manifoldless ADG-gravity, singularities are not thought of as breakdown points of the law of gravity, at least not in any differential geometric sense. Quite on the contrary, *the ADG-formulated Einstein equations are seen to hold over singularities of any kind*. This is not so much a '*resolution*' of singularities in the usual sense of the term, as an '*absorption*' of them in the ADG-gravitational field $(\mathcal{E}, \mathcal{D})$. That is, singularities are incorporated in $\mathbf{A}$ (thus, in effect, they are absorbed in $\mathcal{E}$), in the sense that they are singularities of some functional, generalized coordinate-type of entity in the structure sheaf of generalized arithmetics that *we* choose in the first place to employ in the theory. The aforementioned $\mathbf{A}$-functoriality of the ADG-gravitational dynamics secures that the ADG-gravitational field 'sees through' the singularities carried by $\mathbf{A}$, and the latter in no sense are breakdown *loci* of the differentially (:differential geometrically) represented field law of gravity as a differential equation proper as the manifold and CDG-based analysis of spacetime singularities has hitherto made us believe (Clarke, 1993). Thus, in view of the ADG-generalized PGC and its associated PFR mentioned in the previous section, Einstein's 'non-belief' in singularities can be succinctly justified in ADG-gravity as follows:

> What has nature (here, the physical field of gravity and the law that it defines as a differential equation) to do with coordinates (here, $\mathbf{A}$) and the singularities that they carry? If coordinates are unphysical because they do not partake into the ADG-gravitational dynamics (:$\mathbf{A}$-functoriality of (1)), then so are singularities, since they are inherent in $\mathbf{A}$.

Nevertheless, the general opinion nowadays is that, although gravitational singularities are a problem of classical gravity (GR) long before its quantization becomes an issue, a quantum theory of gravity should, if not remove them completely much in the same way that quantum electrodynamics did away with the unphysical infinities in Maxwell's theory, at least show us a way towards their resolution (Penrose, 2003). We thus turn to some quantum implications of the base

[22] Our emphasis.
[23] Here it is tacitly assumed that a differential manifold $M$ is nothing else but the algebra $\mathcal{C}^{\infty}(M)$ of smooth functions on it (Gel'fand duality).

manifoldless ADG-gravity and how the singularity-absorption into **A** mentioned above may come in handy.

*Towards a 3rd-quantized theory of gravity.* The ADG-theoretic outlook on gravity is field-theoretic *par excellence*. In fact, it is purely 3rd-gauge field-theoretic, as it employs solely the algebraic connection field and there is no external (to the field) geometrical spacetime manifold.

From a geometric (pre)quantization and 2nd (:field) quantization vantage, the (local) sections of $\mathcal{E}$ represent (local) quantum particle (position) states of the field.[24] Moreover, these 'field quanta' obey an ADG-analogue of the spin-statistics connection: extending to vector sheaves Selesnick's bundle-theoretic musings in Selesnick (1983), boson states correspond to sections of *line* sheaves,[25] while fermions are represented by sections of vector sheaves of rank greater than 1.

Parenthetically, it must be noted here that the said representation of (gauge and matter) particle-quanta states as sections of the corresponding $\mathcal{E}$s ties well with the aforesaid incorporation of singularities in **A** (or $\mathcal{E}$), in the following sense: ever since the inception of GR, and subsequently with the advent of QM, it is well reported that Einstein in his unitary field theory program[26] wished to describe the particle-quanta as '*singularities in the field*'. Prophetically, Eddington (1920) anticipated him:

> "... It is startling to find that the whole of dynamics of material systems is contained in the law of gravitation; at first gravitation seems scarcely relevant in much of our dynamics. But there is a natural explanation. *A particle of matter is a singularity in the gravitational field*,[27] and its mass is the pole-strength of the singularity; consequently *the laws of motion of the singularities must be contained in the field-equations*,[28] just as those of electromagnetic singularities (electrons) are contained in the electromagnetic field-equations..."

By absorbing the singularities into **A**, by identifying quantum-particle states as sections of $\mathcal{E}$ (*ie*, in effect of **A**!), and by the **A**-functoriality of the ADG-gravitational dynamics, we have a direct realization of Eddington's anticipation above: *the particle-quanta co-vary with the field-law itself*. In a strong de Broglie-Bohmian sense, the connections are the '*guiding fields*' of their particles: they embody them and carry them along the dynamics (:field equations) that they define.

The upshot of all this is that, due to the external spacetime manifoldlessness of the theory, the quantum perspective on ADG-gravity:

[24] Indeed, $\mathcal{E}$ may be thought of as the associated (:representation) sheaf of the principal group sheaf $\mathcal{A}ut\mathcal{E}$ of field automorphisms.

[25] Vector sheaves of rank 1.

[26] Which, let it be noted here, was intended to 'explain away' QM altogether.

[27] Our emphasis.

[28] Again, our emphasis.

- May be coined 3rd-quantum field theory.[29] *In toto*, *QG from the ADG-perspective is a 3rd-quantum, 3rd-gauge field theory*.
- Since the ADG-gravitational field is an external spacetime unconstrained gauge system, there is also *prima facie* no problem in defining (gauge invariant) observables in (vacuum) Einstein gravity (Torre, 1993), or a (physical) inner product (:physical Hilbert space); while no problem of time arises either, since Diff$(M)$ is absent from the theory from the very start (Isham, 1993; Torre, 1994).[30]
- In a possible covariant (:path integral) quantization of ADG-gravity, the physical configuration space is the moduli space of the affine space $\mathsf{A}$ of $\mathbf{A}$-connections, modulo the field's gauge auto-transformations in $\mathcal{A}ut\mathcal{E}$. Here too, since Diff$(M)$ is not present, there should be no problem in finding a convenient measure to implement the said functional integral. Towards this end, and with some new ADG-results in hand (Mallios, 2005c), Radon-type of measures on $\mathsf{A}/\mathcal{A}ut\mathcal{E}$ are currently being investigated. There have been recent QG tendencies to develop differential geometric ideas and a related integration theory on the moduli space of gravitational connections, as for example in Loop Quantum Gravity (LQG) (Ashtekar and Lewandowski, 1995a,b; Smolin, 2004), but advances appear to be stymied by the ever-present background smooth spacetime manifold and its associated Diff$(M)$ (Baez, 1994a,b).
- *There is no quantization of spacetime per se entertained in ADG-gravity, since there is no spacetime to begin with*. Such a spacetime quantization procedure figures prominently in current gauge-theoretic (*i.e.*, connection based) approaches to QG such as LQG, and it is used there to resolve smooth spacetime singularities (Husain and Winkler, 2004; Modesto, 2004). Thus here we have an instance of the aforesaid general anticipation of current QG researchers, namely, that a quantum theory of gravity should remove singularities. Indeed, LQG appears to resolve singularities via spacetime quantization. Again, this must be contrasted against ADG-gravity, where *ab initio* there is no spacetime continuum hence no spacetime quantization either, while singularities are being absorbed in the field law itself; hence, strictly speaking, there is no need for their 'quantum resolution'.

[29] Recently, Petros Wallden brought to the attention of this author that the term '*third quantization*' has already been used in quantum gravity and quantum cosmology research (Strominger, 1991). However, the sense in which we use this term is quite different from that.

[30] All these problems are encountered in the manifold (and CDG) based canonical approaches to QG, in which the gravitational field is viewed as a spacetime constrained gauge system and Diff$(M)$ represents those so-called primary space-time constraints (:in a canonical $3+1$-split smooth spacetime manifold setting, the primary constraints are the 3-spatial diffeos and the Hamiltonian time-diffeo resulting in the celebrated Wheeler-de Witt equation satisfied by physical states).

- Last but not least comes the issue of the formulation of a manifestly *background independent* non-perturbative QG (Álvarez, 2004; Ashtekar and Lewandowski, 2004; Smolin, 2004). Normally, 'background independence' means '*background geometry (:metric) independence*'. ADG-gravity is explicitly background metric independent, since no metric is involved in the theory (*i.e.*, the aforementioned **A**-metric has no physical significance—it is not a dynamical variable—in the theory).[31] Furthermore, unlike the current connection based approaches to QG, which vitally rely on a background smooth manifold for their differential geometric concepts and constructions, *ADG-gravity is manifestly background spacetime manifold independent.*

Thus, in view of all the virtues of ADG-gravity above, one is tempted to ask the following couple of questions:

- In the guise of (1), *don't we already possess a quantum version of the (vacuum) Einstein equations?*

and concomitantly:

- Since not only a background metric, but also a background spacetime (manifold) is *not* involved in the theory, does the need arise to *quantize spacetime itself*?

The immediate reply is '*yes*' and '*no*', respectively.

*The future in a nutshell: QG in a topos.*. The last paragraph in the present section is concerned with the possibility of formulating ADG-theoretically QG in a topos. A topos is a special type of category that can be interpreted both as an abstract 'pointless space' and as a 'logical universe of variable mathematical entities'. In a topos, geometry and logic are unified (MacLane and Moerdijk, 1992). Thus, the basic intention here is to organize the sheaves involved in ADG-gravity into a topos-like structure in which deep logico-geometrical issues in QG can be addressed. A mathematical byproduct of such an investigation would be to link ADG with the topos-theoretic Synthetic Differential Geometry (SDG) of Kock and Lawvere (Kock, 1981; Lavendhomme, 1996), which in turn has enjoyed various applications so far to classical and quantum gravity (Butterfield and Isham, 2000; Grinkevich, 1996; Guts, 1991, 1995a,b; Guts and Demidov, 1993; Guts and Grinkevich, 1996; Isham, 2003a). In this respect, of purely mathematical interest would be to compare and try to bring together under a topos-theoretic setting the principal notion of both ADG and SDG—that of *connection* (Kock, 1981; Kock and Reyes, 1979; Lavendhomme, 1996; Mallios, 1988, 1998a; Vassiliou, 1994). In the context of a finitary, causal and quantal version of Lorentzian gravity formulated in ADG-terms (Mallios and Raptis, 2001, 2002, 2003, 2005; Raptis,

[31] It is an optional, auxiliary structure externally (to the field $\mathcal{D}$) imposed by the experimenter (:'observer' or 'measurer'); much like **A** itself.

2005a), this enterprize (with a Grothendieck topos twist closely akin to a recent approach to quantum geometry and QG coined 'Causal Site Theory' (Christensen and Crane, 2004))[32] has already commenced (Raptis, 2005b).[33]

Another categorical approach to QG which ADG-gravity could in principle be related to is the recent '*Quantizing on a Category*' (QC) general mathematical scheme due to Isham (2003b, 2004a,b, 2005). The algebraico-categorical QC is closely akin to ADG both conceptually and technically, having affine basic motivations and aims. QC's main goal is to quantize systems with configuration (or history) spaces consisting of 'points' having internal (algebraic) structure. The main motivation behind QC is the failure of applying the conventional quantization concepts and techniques to 'systems' (*e.g.*, causets or spacetime topologies) whose configuration (or general history) spaces are far from being structureless-pointed differential manifolds. Isham's approach hinges on two innovations: first it regards the relevant entities as objects in a category, and then it views the categorical morphisms as abstract analogues of momentum (derivation maps) in the usual (manifold based) theories. As it is the case with ADG, although this approach includes the standard manifold based quantization techniques, it goes much further by making possible the quantization of systems whose 'state' spaces are not smooth continua.

Indeed, there appear to be close ties between QC and ADG-gravity—ties which ought to be looked at closer. *Prima facie*, both schemes concentrate on evading the (pathological) pointed differential manifold—be it the configuration space of some classical or quantum physical system, or the background spacetime arena of classical or quantum (field) physics—and they both employ 'pointless', categorico-algebraic methods. Both focus on an abstract (categorical) representation of the notion of derivative or derivation: in QC, Isham abstracts from the usual continuum based notion of vector field (derivation), to arrive at the categorical notion of arrow field which is a map that respects the internal structure of the categorical objects one wishes to focus on (*e.g.*, topological spaces or causets); while in our work, the notion of derivative is abstracted and generalized to that of an algebraic connection, defined categorically as a sheaf morphism, on a sheaf of suitably algebraized structures (*e.g.*, causal sets, or finitary topological spaces and the incidence algebras thereof representing quantum causal sets, as in the finitary version of ADG-gravity (Mallios and Raptis, 2001, 2002, 2003; Raptis, 2005a,b)).

## 4. EPILOGUE: GENERAL CLOSING REMARKS

In this epilogue we would first like to discuss whether it is still reasonable to believe that we can use differential geometric ideas in the quantum deep, that

[32] A categorical generalization of the 'Causal Set Theory' of Bombelli *et al.* (1987); Sorkin (1995, 1997, 2003).

[33] Anticipatory works of such an enterprize are Raptis (1996, 2001, 2003).

is, in the QG domain. Then, we would like to conclude this paper by continuing the general theme of the prologue, namely, that QG research is in need of new concepts, new mathematics, and a novel way of philosophizing about them.

*Still use differential geometry in QG?*. Although the general feeling nowadays among theoretical physicists (and in particular, 'quantum gravitists') is that below a so-called Planck length-time ($\ell_P$-$t_P$),[34] where quantum gravitational effects are supposed to become significant, the space-time continuum (:manifold) should give way to something more reticular (:discrete) and quantal, CDG-ideas and technology still abound in current QG research. Consider for instance the manifold based CDG used in all its glory in the canonical and covariant approaches to QG (*e.g.*, LQG (Ashtekar and Lewandowski, 1995a,b)), or the higher-dimensional (real analytic or holomorphic) manifolds (*e.g.*, Riemann surfaces, Kähler manifolds, Calabi-Yau manifolds, supermanifolds, *etc.*) engaged in (super)string theory research, or even the so-called noncommutative differential spaces that Connes' Noncommutative Differential Geometry propounds (Connes, 1994, 1998; Kastler, 1986), which are still, deep down, differential manifolds in the usual sense of the term. *In toto*, smooth manifolds and CDG are still well and prosper in QG.

A few people, however, have aired over the years serious doubts about whether the spacetime continuum and, *in extenso*, the CDG that is based on it, could be applied *at all* in the QG domain. Starting (in chronological order) with Einstein, then going to Feynman, the doubts reach their climax in Isham's categorematic '*no-go of differential geometry in QG*' below:

> "... You have correctly grasped the drawback that the continuum brings. If the molecular view of matter is the correct (appropriate) one; *i.e.*, if a part of the universe is to be represented by a finite number of points, then the continuum of the present theory contains too great a manifold of possibilities. I also believe that this 'too great' is responsible for the fact that our present means of description miscarry with quantum theory. *The problem seems to me how one can formulate statements about a discontinuum without calling upon a continuum space-time as an aid; the latter should be banned from theory as a supplementary construction not justified by the essence of the problem—a construction which corresponds to nothing real. But we still lack the mathematical structure unfortunately.*[35] How much have I already plagued myself in this way [of the manifold]!... " (Stachel, 1991)

............................

> "... *The theory that space is continuous is wrong, because we get ... infinities* [ viz. 'singularities'] *and other similar difficulties* ... [ while] *the simple ideas of geometry, extended down to infinitely small, are wrong*[36] ... " (Feynman, 1992)

............................

[34] $\ell_P = \sqrt{\frac{G\hbar}{c^3}} = 1.6 \times 10^{-33}$ cm; $t_P = \sqrt{\frac{G\hbar}{c^5}} = 5.3 \times 10^{-44}$ s.

[35] Our emphasis.

[36] Our emphasis throughout.

"...*At the Planck-length scale, differential geometry is simply incompatible with quantum theory* ... [so that] *one will not be able to use differential geometry in the true quantum-gravity theory*[37] ..." (Isham, 1991)

Isham's remarks are shrewd, critical and iconoclastic:

*CDG and the classical* $\mathcal{C}^{\infty}$*-smooth manifold model of spacetime supporting its constructions 'miscarry with'* (to use Einstein's expression above) *quantum theory, and it will therefore be of no import to QG research.*

On the other hand, and this is one of the basic aftermaths of our work, from an ADG-theoretic point of view it is not exactly that differential geometric ideas cannot be used in the quantum regime—as if the intrinsic differential geometric mechanism (which in its essence is of an algebraic nature) fails in one way or another when applied to the realm of QG—but rather that when that mechanism is geometrically effectuated or implemented (represented) by the (cartesian mediation in the guise of the smooth coordinates of the) background $\mathcal{C}^{\infty}$-smooth spacetime manifold as in CDG, then all the said problems (:singularities, unphysical infinities, $\mathrm{Diff}(M)$-related pathologies) crop up and are insurmountable (always within the confines of, *i.e.*, with the concepts and the methods of, the theoretical framework of the manifold based Analysis).

Thus, to pronounce this subtle but crucial from the ADG-perspective difference, we maintain that

the second part of Isham's quotation above should also carry the adjective '*classical*' in front of '*differential geometry*', and read: '*one will not be able to use classical differential geometry*' (or equivalently, a geometrical base differential spacetime manifold) '*in the true quantum-gravity theory*'.

*In summa*, the aforesaid subtle distinction hinges on the physical non-existence of a background geometrical smooth spacetime manifold, *not* of the inapplicability of the *essentially algebraic mechanism of differential geometry*, which can still be retained and applied to QG research. Metaphorically speaking, ADG-gravity has shown us a way *not* to throw away the baby (:the invaluable algebraic differential geometric mechanism) together with the bath-water (:the base smooth spacetime manifold). The 'icon' (or perhaps better, the 'idol') that Isham's iconoclastic words ought to cut out of physics once and for all is the background geometrical spacetime manifold and *not* the invaluable differential geometric machinery which CDG has so far misled us into thinking that is inextricably tied to the base manifold.

To summarize, in the background geometrical spacetime manifoldless ADG-gravity, all the classical and quantum gravity problems we mentioned in the previous section, which are all due to the base $M$, its $\mathrm{Diff}(M)$ and, *in extenso*,

[37] Our emphasis.

to the CDG that is based on the latter, simply disappear—*i.e.*, they become non-problems. Thus, ADG does not solve these puzzles; it simply cuts the Gordian knot that they present us within the CDG-framework. This is analogous to how Wittgenstein Wittgenstein (1980) maintained that philosophical problems could be solved: simply by changing perspective—ultimately, by changing theoretical framework:

> "... The solution of philosophical problems can be compared with a gift in a fairy tale: in the magic castle it appears enchanted and if you look at it outside in daylight it is nothing but an ordinary bit of iron (or something of the sort)[38] ... "

Indeed, problems in GR like that of singularities and Einstein's hole argument (Stachel, 1987, 1989, 1993b, 2002), as well as the problem of time and that of observables in QG, look formidable (in fact, insuperable!) when viewed and tackled via the manifold based CDG—ultimately, when we are bound by "*the golden shackles of the manifold*" (Isham, 1991). However, under the light of ADG, '*gold looks nothing but an ordinary bit of iron*'. Furthermore, much in the same way that Wittgenstein (1956) contended that

> "... Our task is, not to discover new calculi, but to describe the present situation in a new light... "

our ADG-framework (and, as a result, ADG-gravity), does not purport to be some kind of new Differential Calculus (and, accordingly, ADG-gravity a new theory of gravitation); it simply goes to show that most (if not all!) of the differential geometric mechanism 'inherent' in CDG can be articulated entirely algebraically, without the cartesian mediation of a background geometrical (spacetime) manifold (with all the supposedly physical pathologies that the latter is pregnant to). In addition, it goes without saying that if the base geometrical $M$ has to go, so must the geometrical (spacetime) interpretation of the theory (:GR).[39]

For after all, Einstein too, overlooking the great success that the geometrical spacetime manifold based GR enjoyed during his lifetime, insisted that:

> "... Time and space are modes by which *we think*, not conditions in which we live" (Einstein, 1949)[40] ... "[the spacetime continuum] corresponds to nothing real" (Stachel, 1991)..., [but perhaps more importantly, that] "[Quantum theory] *does not seem to be in accordance with a continuum theory, and must lead to an attempt to find a purely algebraic theory for the description of reality. But nobody knows how to obtain the basis of such a theory*" (Einstein, 1956).

Indeed, we are tempted to say that when Einstein was talking about "... *concepts which have proven useful for ordering things easily assume so great an authority over us, that we forget their terrestrial origin and accept them as unalterable facts.*

[38] Our emphasis throughout.
[39] Of course, it now behooves us to answer to the question posed at the end of section 2.
[40] This quotation can also be found in Anastasios Mallios' contribution to this volume.

*They then become labelled as 'conceptual necessities', 'a priori situations', etc.*" in the quotation we saw in the introduction, he was 'subconsciously' referring to the *a priori* concept (and use by CDG-means) of the spacetime continuum in GR. Moreover, again to emulate Einstein's concluding words in that quotation, we believe that

> *the road of progress in QG has been blocked for a long period by our erroneous insistence on the 'physicality' of the background geometrical spacetime continuum.*

Parenthetically, and on more general grounds, let it be stressed here that Einstein, during his later years, went as far as to insist that (and we quote him indirectly via Peter Bergmann from Bergmann (1979)):[41]

> "... geometrization of physics is *not* a foremost or even a meaningful objective... "

Thus, we see that Einstein towards the end of his life tended to leave behind 'geometry' and take on 'algebra' *vis-à-vis* the quantum domain.

Lately, Einstein's words for a purely algebraic description of physical phenomena in the quantum deep in the penultimate quotation above, have found fertile ground as there have been tendencies towards a purely algebraic theoresis of QG. Ten years ago, Louis Crane asked characteristically in the very title of a paper of his Crane (1995):

> *"Clock and category: is quantum gravity algebraic?"*

The purely algebraico-categorical ADG-gravity appears to answer to it affirmatively, and what's more, in a *background spacetime manifoldless differential geometric setting*, in spite of Isham's doubts and reservations above. May ADG provide the theoretical framework that Einstein was (and some of us still are nowadays!) looking for in our journey towards QG. However, even if that does not turn out to be the case in the end, at least we will have in our hands an entirely algebraic (re)formulation of differential geometry—a novel framework pregnant with new concepts, new principles, new techniques, and new theoretical terms. Following Wallace Stevens' (1990) dictum, that:

> "... Progress in any aspect is a movement through changes in terminology... "

we believe it is worth trying to move towards our QG destination through the ADG-path. For in any case, from the novel viewpoint of ADG, we may at least be able to see 'old' and 'stale', but nevertheless persistent, problems (like for example the $\mathcal{C}^{\infty}$-singularities of the manifold and CDG based GR) with 'new' and 'fresh' eyes.[42] Schopenhauer's words from Schopenhauer (1970) immediately spring to mind here:

[41] This quotation can also be found in Anastasios Mallios' contribution to this volume.

[42] And that's no small feat, if we consider Wittgenstein's remarks from Wittgenstein (1980) quoted earlier.

> "*...Thus, the task is not so much to see what no one has yet seen, but to think what nobody yet has thought about that which everybody sees*[43] *...*"

Let us now pick the argument from where we left it earlier, when the problem of gravitational singularities was discussed under the prism of the background spacetime manifoldless ADG-gravity, and comment on the closely related problem of the unphysical infinities associated with those singularities, as well as the non-renormalizable infinities appearing in QG when treated as another, manifold based, QFT.

*Whence the unphysical infinities?* There are infinities associated with gravitational singularities, there is no doubt about that. For instance, the curvature of the spherically symmetric Schwarzschild gravitational field of a point-particle of mass $m$ diverges as $m^2/r^6$ as one approaches it ($r \longrightarrow 0$); moreover, there is no analytic extension of the Schwarzschild spacetime manifold so as to include the singular *locus* $m$ with the other regular points of the manifold (Clarke, 1993). In contradistinction to the exterior Schwarzschild singularity at $r = 2m$ (:horizon) which has been branded a virtual, coordinate singularity, the interior $r = 0$ one is thought of as a true singularity, with physical significance. Nevertheless, it is altogether hard to believe that there are actually physically meaningful infinities in Nature.

As noted earlier, many researchers hoped (and still do!) that QG will remove singularities in the same way that QED removed the Maxwellian infinities. Thus, perturbative QG, by emulating the other quantum gauge theories of matter, initially regarded QG as another QFT (on a flat Minkowski background!) and evoked the (arguably *ad hoc*) process of renormalization to remove gravitational infinities. It soon failed miserably, because of the dimensionful gravitational coupling constant. Theoretical physicists are people of resourcefulness, strong resolve and stout heart, thus they evoked (or 'better', they introduced by hand!) extra dimensions, extra fields to occupy them and extra symmetries between those extra fields (*e.g.*, supergravity and supersymmetric string theories) in order to 'smear' the offensive *loci*, much like one blows up singularities in algebraic geometry. The singular interaction point-vertices of the Feynman diagrams of the meeting propagation lines of the point-particles of QFT were smeared and smoothened out by world-tubes of propagating closed strings, being 'welded' smoothly into one another at the interaction sites. However, infinities, although tamed a bit, are still seen to persist galore (never mind the grave expense of theoretical economy that accompanies the introduction of more and more in principle unobservable fields and their particle-quanta).[44]

[43] Emphasis is ours.

[44] Of course, this blatant violation of Occam's razor is not necessarily bad by itself, as at least it keeps the experimentalists busy (and quiet!) designing experiments to look for the 'predicted' extra particles (*e.g.*, the superpartners of the known particles), whose existence appears to be mandated by theory.

At the same time, people from the non-perturbative QG camp soon realized that non-renormalizability is not a problem in itself if one takes into consideration that QG, as opposed to the other quantum gauge forces of matter, has associated with it a fundamental space-time scale—the Planck length-time—which as noted earlier is an expression involving the fundamental constants of the three theories that are supposed to be merged into QG: $G$ from (Newtonian) gravity, $c$ from relativity, and $\hbar$ from quantum mechanics. The Planck scale can then be thought of as prohibiting *in principle* the integration down to infinitely small spacetime distances; or dually in the perturbation series/integrals, up to infinite momenergies. Non-perturbative QG fundamentally assumes that spacetime is inherently cut-off (:'regularized') by the Planck scale, so that below it the continuum picture should be replaced by something more discrete and quantum.

All this is well known and good. The infinities have not only kept us occupied for a while, but they have provided us with a wealth of new ideas and techniques in our struggle and strife to remove them (*e.g.*, anomalies, spontaneous symmetry breaking, phase changes, catastrophes and other critical phenomena, as well as the renormalization group technology that goes hand in hand with them, *etc.* (Jackiw, 2000)). However, their stubborn persistence makes us still abide by our main thesis here: it is indeed the background smooth spacetime continuum, accommodating uncountably infinite degrees of freedom of the fields which are modelled after smooth functions on it (or $\otimes_{C^\infty_M}$-tensors thereof), that is responsible for all those pestilential infinities. We must therefore give up *in principle* the spacetime continuum (:manifold) and the usual Analysis (Calculus or CDG) based on it, because they appear to miscarry in the QG deep. In this line of thought we can metaphorically paraphrase Evariste Galois':

> "*Les calcules sont impracticables*",[45]

and add that *the Differential Calculus, when effectuated via the background geometrical spacetime continuum, is an obstacle rather than a boon to QG research.* In turn, this reminds us of Richard Feynman calling the usual differential geometry "*fancy schmanzy*", doubting the up-front geometrical interpretation of GR, and opting instead for a combinatory-algebraic (diagrammatic-relational) scheme along QFTheoretic lines for its quantization (Feynman, 1999).[46] Of course, Feynman's unsuccessful attempt at quantizing gravity by applying the perturbative-diagrammatic technology of QED is well documented.

[45] "*Calculations are impractical*".

[46] See especially the forward, titled "*Quantum Gravity*", written by Brian Hatfield, giving a brief account of Feynman's approach to QG. Hatfield argues there that Feynman not only felt that the (differential) geometrical interpretation of gravity 'gets in the way of its quantization', but also that it masks its fundamental gauge character.

At the same time, from the ADG-gravitational perspective we cannot accept the non-perturbative QG's thesis that there is a fundamental spacetime scale in Nature either, simply because there is no spacetime in Nature to begin with. From our viewpoint, in a Leibnizian sense, *'spacetime' is the (dynamical) objects that comprise 'it'; that is, the (dynamical) fields*. Accepting the existence of a fundamental scale in Nature, above which Einstein's equations hold, but below which the latter break down and another set of equations (:those of the QG we are supposed to be after) are in force, is analogous to accepting singularities as *physical* entities. They both violate the universality of Physical Law, and undermine the unity and autonomy of the gravitational field.

That our calculations are plagued by infinities is more likely because the usual Differential Calculus that *we* employ is inextricably tied to a geometrical base spacetime continuum that *we* assume up-front in the theory. Our manifold based Analysis invites infinities by allowing for infinitary processes (of divergence) relative the base topological continuum. On the other hand, *there is no infinity in algebra*, and our purely algebraic ADG-gravity suffers from no such unphysical pathologies. It would be weird, or indeed comical(!), to even try to fathom what would the meaning of the notion of 'singularity' be in a purely 'pointless' and 'space(tile)less' algebraico-categorical setting like ours. For example, an attempt at the following analogy produces funny thoughts:

> Does a singularity bend (or break!) the categorical arrows (:connection field morphisms) in ADG-gravity in a way analogous to how a point-electron is geometrically envisaged to distort the Faraday lines of force of the electromagnetic field in its vicinity? Then, *mutatis mutandis* for the gravitational field lines of force strongly focusing towards a point-mass, as in the case of the interior Schwarzschild (black hole) singularity.

*New theoretical-mathematical framework for QG.*. To connect this epilogue back to the prologue like the proverbial tail-biting serpent, in QG research the glaring absence of comprehensive experiments and thus of reliable and concrete experimental data to support and constrain theory-making is, at least from a mathematical viewpoint, quite liberating. The tentative, transient and speculative nature of the field invites virtually unrestrained conceptual imagination, mathematical creativity and wild philosophical wandering.

Even that most austere and critical of all 20th century theoretical physicists, Wolfgang Pauli, said about the prospect of quantizing the gravitational field (Pauli, 1994):

> "... Every theoretical possibility is a potential route to success... [however, in this field] only he who risks has a chance to succeed... "[47]

Abiding by John Wheeler's dictum that '*more is different*', the plethora of (mathematical) approaches to QG are more than welcome (even if we coined it the

[47] See also Feynman's quotation in the introductory paper to this volume.

seemingly derogatory '*zoo*' in the prologue!), under the proviso that every now and then unifying efforts are made to patch together the mosaic of approaches to QG into a single—or at least to a regular—pattern tapestry. This can be achieved for example by occasionally leaving the worm's eye-view—as it were, the 'local', nitty-gritty problems and technical calculations of each individual approach—and by trying to attain a 'global' conceptual, bird's eye-view of the field; one that at least tries to make 'dictionary correspondences', in both conceptual and technical jargon, between different approaches. For Nature is economical, and so must be our theories of Her—if not in (mathematical) technicalities, at least conceptually.

On the other hand, Paul Dirac, more than 70 years ago (Dirac, 1931), implored us to apply all our existing mathematical arsenal, and even to invent and create new mathematics in order to tackle the outstanding theoretical physics problems of the last century—QG arguably being *the* central one that stubbornly resists (re)solution in our times:[48]

> "... The steady progress of physics requires for its theoretical foundation a mathematics that gets continually more advanced. This is only natural and to be expected. What, however, was not expected by the scientific workers of the last century was the particular form that the line of advancement of the mathematics would take, namely, it was expected that the mathematics would get more complicated, but would rest on a permanent basis of axioms and definitions, while actually the modern physical developments have required a mathematics that continually shifts its foundation and gets more abstract.... It seems likely that this process of increasing abstraction will continue in the future and that advance in physics is to be associated with a continual modification and generalization of the axioms at the base of mathematics rather than with logical development of any one mathematical scheme on a fixed foundation.
> There are at present fundamental problems in theoretical physics awaiting solution [...][49] the solution of which problems will presumably require a more drastic revision of our fundamental concepts than any that have gone before. Quite likely these changes will be so great that it will be beyond the power of human intelligence to get the necessary new ideas by direct attempt to formulate the experimental data in mathematical terms. The theoretical worker in the future will therefore have to proceed in a more indirect way. *The most powerful method of advance that can be suggested at present is to employ all the resources of pure mathematics in attempts to perfect and generalise the mathematical formalism that forms the existing basis of theoretical physics, and after each success in this direction, to try to interpret the new mathematical features in terms of physical entities*[50] ..."

At the same time, however, there is this nagging little voice at the back of every theoretical physicist's mind cautioning her about the *New Maths Version of Murphy's Law*, maintaining that

[48] Quote borrowed from fairly recent paper by Faddeev (2000).

[49] At this point Dirac mentions a couple of outstanding mathematical physics problems of his times, which are hereby omitted.

[50] Our emphasis.

> *whenever there is a 50–50 chance that a new mathematical theory applies to physics successfully, 9 times out of 10 it turns out to fail,*[51]

notwithstanding Eugene Wigner's '*unreasonable effectiveness of mathematics*'. In turn, this further evokes forebodings of scepticism and fear, reminding her of Pauli's (in)famous remark that "*this theory is so bad, it's not even wrong*".

Nevertheless, it is the main position of this author that such reservations and phobias have to be put aside in the dawn of the new millennium, for in the end they only present inertia to, and create an attitude of pessimism (invariably resulting to indolence) in the development of theoretical physics. We have to be innovative, adventurous and unconventional, perhaps even iconoclastic,[52] not only about our technical-mathematical machinery, but also about the conceptual and philosophical underpinnings of our fundamental theories of Nature—with QG in particular, since it is arguably the deepest of them all. Gerard 't Hooft put it succinctly in 't Hooft (2001):

> "... The problems of quantum gravity are much more than purely technical ones. They touch upon very essential philosophical issues... "

Thus, we should not inappreciably pass-by this unique opportunity that QG is offering us: to bring together Physics and Philosophy, thus reinstate the luster of '*Naturphilosophie*' that theoretical physics seems to have lost in the last century, predominantly due to its focusing on technical (:mathematical) formalism, atrophizing at the same time important conceptual/interpretational issues.

Ultimately, we should not be afraid of making mistakes, or fear that our theories will come short of describing Nature completely, because anyway, on the one hand the maths is our own free intellectual creation[53] (thus, we can take responsibility for their shortcomings and blemishes, and rectify them when necessary), while on the other, Physis is almost *de facto* wiser than us. This simply goes to show that theoretical physics is a never ending quest, and thus that our theories are in a constant process of revision, refinement and extension.

To close this epilogue the way we started it, as Faddeev maintains in Faddeev (2000) motivated by Dirac's remarks above, theoretical/mathematical physics cannot—in fact it *should not*—rely anymore on experiment for its progress. It should become more and more autonomous, more and more abstract, as well as versatile and wide ranging. Once again, the tried and tested age-old virtues of conceptual simplicity, mathematical economy and beauty—virtues that are trademarks in the celebrated works of such giants as Einstein and Dirac—can be called

[51] A watered down version of what David Finkelstein has coined the '*mathetic fallacy*' in theoretical physics (private communication).
[52] See opening paper in this issue.
[53] Recall from the quotes given above Einstein referring to the (mathematical) concept of the (space-time) continuum as a '*mode by which we think*', as well as his warning us in general not to forget the '*terrestrial origin*' of various concepts, no matter how useful they may have been in the past.

to guide us in our theoretical physics (ad)ventures through our presumed 'subject': Physis.[54] And we can rest assured that these virtues shall safeguard us from 'mathematically arbitrary' theory-making.

> After all, it is well known that when the solar eclipse results were due back from Arthur Eddington's 1919 Cape Town expedition, in Berlin Max Planck could not go to sleep in anticipation and excitement about whether GR would be experimentally (:observationally) vindicated; or on the contrary, whether it would fail to deliver in the end. Einstein on the other hand reportedly went to bed by eight o'clock. . . .

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[54] Of course, if anything, *we* are the subjects of Nature, not the other way round. Hence the quotation marks.

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